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Compositeness Test at the FMC with Bhabha Scattering

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Abstract. It is possible that quarks and/or leptons have substructure that will become manifest at high energies. Here we investigate the limits on the muon compositeness scale that could be obtained at the First Muon Collider using Bhabha scattering. We study this limit as a function of the collider energy and the angular cut imposed by the detector capability.

I INTRODUCTION

The presence of three generations of quarks and leptons, apparently identical except for mass, strongly suggests that they are composed of still more fundamental fermions. It is clear that, if substructure exist, the associated strong interaction energy scale Λ must be much greater than the quark and lepton masses. Long ago, 't Hooft figured out how interactions at high energy could produce essentially massless composite fermions: the answer lies in unbroken chiral symmetries of the underlying fermions and confinement of their new strong nonabelian gauge interactions [1]. There followed a great deal of theoretical effort to construct a realistic model of composite quarks and leptons (see, e.g., Ref. [2]) which, while leading to valuable insights on chiral gauge theories, fell short of its main goal.

It was pointed out that the existence of quark and lepton substructure will be signalled at energies well below Λ by the appearance of four-fermion "contact" interactions which differ from those arising in the standard model [3,4]. These interactions are induced by the exchange of bound states associated with the new gauge interactions. The main constraint on their form is that they must be $SU(3) \otimes SU(2) \otimes U(1)$ invariant because they are generated by forces operating at or above the electroweak scale. These contact interactions are suppressed by $1/\Lambda^2$, but the coupling parameter of the exchanges—analogous to the pion-nucleon and rho-pion

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couplings—is not small. Compared to the standard model, contact interaction amplitudes are of relative order $s/(\alpha\Lambda^2)$, where $\sqrt{s}$ is the center of mass energy of the process taking place and α the coupling constant of the standard model interaction. The appearance of $1/\alpha$ and the growth with s make contact-interaction effects the lowest-energy signal of quark and lepton substructure. They are sought in jet production at hadron and lepton colliders, Drell-Yan production of high invariant mass lepton pairs, Bhabha scattering, $e^+e^- \to \mu^+\mu^-$ and $\tau^+\tau^-$ [5], atomic parity violation [6], and polarized Möller scattering [7]. Hadron collider experiments can probe values of Λ from the 2–5 TeV range at the Tevatron to the 15–20 TeV range at the LHC (See Refs. [4,8]).

Here, we will study in some details one specific example for the First Muon Collider (FMC): the constraint that can be imposed on the scale of muon compositeness by measuring Bhabha scattering. The specific form for the muon contact interaction is presented in Section II. (All the results presented here are also applicable to electron compositeness at e^+e^- colliders with same energy and luminosity.)

CELLO at PETRA with a center of mass energy, $\sqrt{s}$, of 35 GeV and an integrated luminosity, $\mathcal{L}$, of 86 pb^{-1} was able to put a lower limit on the (electron) compositeness scale of the order of 2-4 TeV, depending on the specific model for compositeness [9]. This is about the same reach as the current Tevatron reach. This clearly show the potential for lepton colliders to probe compositeness; they have an enormous reach.

In section III, we study the reach versus the energy of the FMC, with the corresponding luminosity chosen for this workshop. We also study the effect of the angular cut on the reach. It is important to study that effect because large amount of radiation close to the beam will limit the capability of the detectors outside the central region.

II   MUON COMPOSITENESS

We assume the muon has a substructure. For collider energy below the scale associated with this new structure, the effect can be parametrized by a four fermions interaction. Here, we use the flavor-diagonal, helicity-conserving contact interaction proposed by E. J. Eichten, K. Lane and M. E. Peskin [3]:

$$\mathcal{L} = \frac{g^2}{2\Lambda^2} \left[\eta_{LL} \; j_L j_L + \eta_{RR} \; j_R j_R + \eta_{LR} \; j_L j_R \right], \tag{1}$$

j_L and j_R are the left-handed and right-handed currents and Λ the compositeness scale. The coupling constant, $\alpha_{\text{new}} \equiv \frac{g^2}{4\pi}$, is assumed to be strong and set to one. By convention, the η have magnitude one.

The unpolarized cross section at lowest order, including the γ and Z exchange (s and t channel) and the contact interaction from Eq. 1 can be written in the following form, see Ref. [9]:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} \left[4B_1 + B_2(1 - \cos\theta)^2 + B_3(1 + \cos\theta)^2 \right] \qquad (2)$$

where

$$B_1 = \left(\frac{s}{t}\right)^2 \left| 1 + (g_V^2 - g_A^2)\, \xi + \frac{\eta_{RL}\, t}{\alpha\Lambda^2} \right|^2, \qquad (3)$$

$$B_2 = \left| 1 + (g_V^2 - g_A^2)\, \chi + \frac{\eta_{RL}\, s}{\alpha\Lambda^2} \right|^2, \qquad (4)$$

$$B_3 = \frac{1}{2}\left| 1 + \frac{s}{t} + (g_V + g_A)^2\left(\frac{s}{t}\xi + \chi\right) + \frac{2\eta_{LL}\, s}{\alpha\Lambda^2} \right|^2$$
$$+ \frac{1}{2}\left| 1 + \frac{s}{t} + (g_V - g_A)^2\left(\frac{s}{t}\xi + \chi\right) + \frac{2\eta_{RR}\, s}{\alpha\Lambda^2} \right|^2, \qquad (5)$$

$$\chi = \frac{G_F}{2\sqrt{2}}\frac{M_Z^2}{\pi\alpha}\frac{s}{s - M_Z^2 + iM_Z\Gamma}, \qquad (6)$$

and

$$\xi = \frac{G_F}{2\sqrt{2}}\frac{M_Z^2}{\pi\alpha}\frac{t}{t - M_Z^2 + iM_Z\Gamma}. \qquad (7)$$

α is the usual fine structure constant, θ the scattering angle between the incoming and outgoing muon, $t = -s/2(1 - \cos\theta)$, g_V and g_A the vector and axial vector coupling constant, M_Z and Γ the mass and width of the Z, and G_F the Fermi constant. We will consider four typical models: LL couplings ($\eta_{LL} = \pm 1$, $\eta_{RR} = \eta_{LR} = 0$), RR couplings ($\eta_{RR} = \pm 1$, $\eta_{LL} = \eta_{LR} = 0$), VV couplings ($\eta_{LL} = \eta_{RR} = \eta_{LR} = \pm 1$), and AA couplings ($\eta_{LL} = \eta_{RR} = -\eta_{LR} = \pm 1$). The positive and negative sign indicate the possible constructive or destructive interference between the electroweak (EW) and compositeness contributions.

TABLE 1. Energy of the collider, luminosity, cross section (with $|\cos\theta| < 0.8$), and the expected number of events.

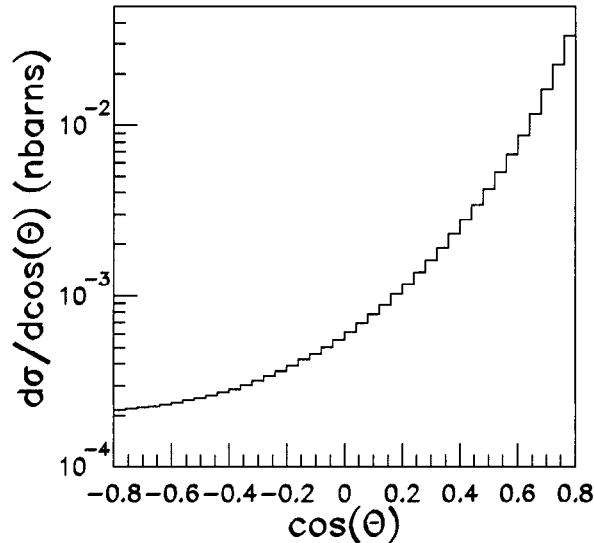| $\sqrt{s}$ (GeV)	$\mathcal{L}(fb^{-1})$	$\sigma(pb)$	N (10^3)
100	.6	125	75
200	1.	34	34
350	3.	11	33
500	7.	5	35

FIGURE 1. $\cos\theta$ distribution at 500 GeV.

III   EXPERIMENTAL BOUNDS

The total cross section for the different energy and luminosity options considered at this workshop is presented in Table 1. The only detector effect included is an angular cut: $|\cos\theta| < 0.8$. No other detector effects were included in this analysis. As is well known, and can be seen in the set of equations presented earlier, the EW cross section decreases proportionally to s (except in the Z resonance region), whereas the interference term is independent of the energy and the pure compositeness term increases with s. This fact combined with the (almost) constant number of events expected as a function of the energy, see Table 1, clearly indicates that the best compositeness limit will come from the highest energy option.

In Fig. 1, the $\cos\theta$ distribution at $\sqrt{s} = 500$ GeV is presented. The typical t-channel, forward peaking is apparent. The $\cos\theta$ distributions at the other energies have the same shape and are therefore not shown.

To show the impact of the compositeness contribution we use the variable Δ, see Ref. [3]:

$$\Delta = \frac{\left(\frac{d\sigma}{d\cos\theta}\right)_{EW+\Lambda} - \left(\frac{d\sigma}{d\cos\theta}\right)_{EW}}{\left(\frac{d\sigma}{d\cos\theta}\right)_{EW}}, \tag{8}$$
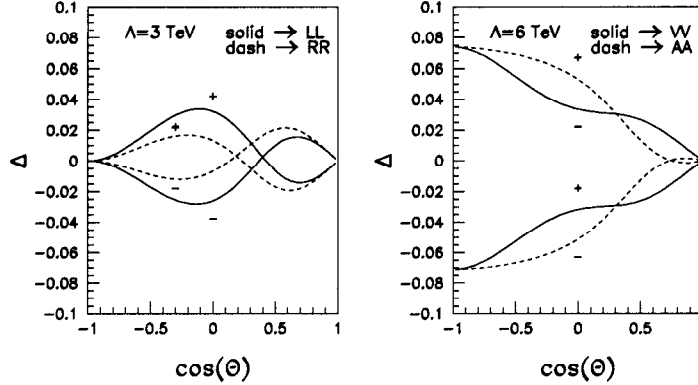
FIGURE 2. The variable Δ versus $\cos\theta$ at $\sqrt{s} = 100$ GeV for the four models, LL, RR, VV, and AA, for the two signs of the η's, indicate by $+$ and $-$ on the plot.

the difference between the theory with and without the compositeness terms, divided by the EW contribution. The $\cos\theta$ distribution of Δ is presented in Fig. 2 for $\sqrt{s} = 100$ GeV. The Λ's were chosen such that the compositeness correction is of the order of 10% compared to the EW contribution. That requires that $\Lambda \simeq 30\sqrt{s}$ for the LL or RR $\left(\frac{s}{\alpha\Lambda^2} \sim .1\right)$ couplings and $\Lambda \simeq 60\sqrt{s}$ for the VV or AA couplings $\left(\frac{s}{\alpha\Lambda^2} \sim .1/4\right)$, there are four interference terms in these latter models, see section II. The results for the four models are shown in Fig. 2 for both sign of the $\eta's$. It is clear that one can get limits on the compositeness scale from the change of the shape of the distribution. Note that in the forward region, in term of sensitivity to the compositeness scale, the smaller change of the shape is compensated by the larger number of events. We also present the distribution at 200 and 500 GeV, in Fig. 3 and 4, respectively. The 300 GeV case is very similar to the 500 GeV case and is not shown.

The next step is to obtain a lower limit on Λ, assuming that the data follow the EW theory. Defining $x = 1/\Lambda^2$, on average (repeating the experiment many times) the central value of x resulting from a χ^2 fit will be zero because the data is assumed to follow the EW theory. For x small enough, the differential cross section is linear in x (the x^2 term is small). Within this approximation the χ^2 is quadratic in x, and the fit can be trivially done. The uncertainty on x, σ_x, is simply given by two times the inverse of the second derivative of χ^2 with respect to x (a constant). We used 20 intervals for the fit, such that the lowest number of events in one bin is still more than 100, which correspond to a maximum 10% statistical uncertainty in each bin. The 95% CL limit on Λ is then obtained from: $\Lambda^2 > 1/(1.64\sigma_x)$. The results are presented in Table 2, for different energies of the muon collider and $|\cos\theta| < 0.8$. As expected the highest energy machine put the strongest constraint on the compositeness scale. The limit that the 4 TeV machine will be able to put (with the luminosity scaled to maintain the number of events constant) is really
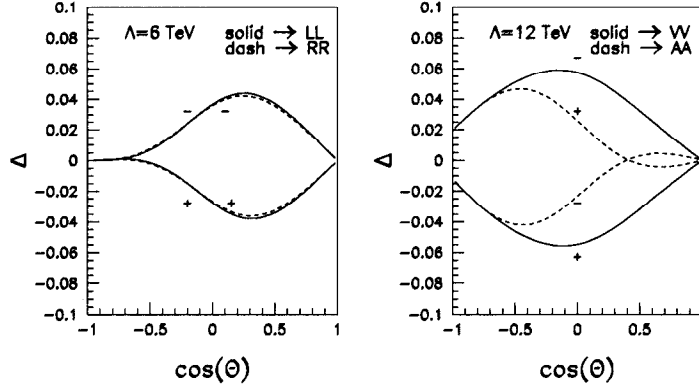
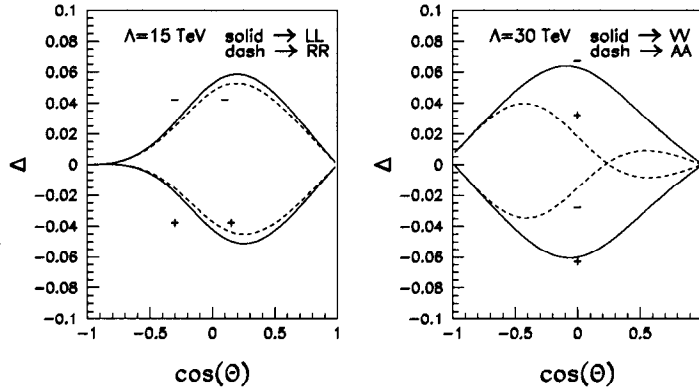**FIGURE 3.** Same as Fig. 2 at 200 GeV, about the LEPII energy.

FIGURE 4. Same as Fig. 2 at 500 GeV.

TABLE 2. 95% CL limits (in TeV) for different energies (in GeV) of the muon collider, we used $|\cos\theta| < 0.8$. We also present the expected LEP limits for which we used $|\cos\theta| < 0.95$.

	LEP(91)	LEP(175)	100	200	350	500	4000
$\mathcal{L}(fb^{-1})$	.15	.1	.6	1.	3.	7.	450.
LL	4.0	5.8	4.8	10	20	29	243
RR	3.8	5.7	4.9	10	19	28	228
VV	6.9	12.	12	21	36	54	435
AA	3.8	7.2	12	13	21	32	263

TABLE 3. 95% CL limits (in TeV) for different angular cuts at $\sqrt{s} = 500$ GeV, $\mathcal{L} = 7fb^{-1}$.

$\lvert\cos\theta\rvert <$	.6	.8	.9	.95
LL	26	29	31	32
RR	24	28	30	30
VV	50	54	56	57
AA	28	32	34	35

impressive. The Λ limits are large enough such that the approximation used (x small) is valid. Because of the approximation used, central value of x equal to zero and the differential cross section linear in x, the limits are independent of the sign of the η. We have only included the statistical uncertainty in this analysis, and the limits presented here should be considered within that context. In particular, the absolute normalization, which is used in this analysis, might be subject to large uncertainties. Our calculated limits for the CELLO case are compatible with their measurements (their central x value is of course non zero). In Table 2, we also have included the expected limit for LEP with the current integrated luminosity (per experiment) and its larger $\cos\theta$ coverage.

In Table 3 we explore the effect of the $\cos\theta$ cut on the 95% CL limit. Although any increase in coverage obviously increases the limit, the improvement between 0.8 and 0.95 is less than 10%. We therefore conclude that the coverage up to 0.8 is adequate for this measurement. It is not necessary to go down very close to the beam to do a very good measurement.

IV   CONCLUSION

We investigated the limits on the muon compositeness scale that could be obtained at the First Muon Collider using the Bhabha scattering process. We considered four typical models for the four-fermion contact terms expected as a low-energy signal for compositeness: LL, RR, VV, and AA couplings.

As expected, the reach increases rapidly with energy. We find that the reach at the 500 GeV FMC is $\Lambda > 30 - 55$ TeV depending on the model. At a future 4 TeV muon collider the range extends to $\Lambda > 230 - 440$ TeV.

The likelihood of limited angular coverage in detectors (because of the unavoidable background of decaying muons) does not appear to poise a severe problem for the study of muon compositeness. We found that an angular coverage corresponding to $\lvert\cos\theta\rvert < 0.8$ is adequate to obtain 90% of the full reach in the compositeness scale Λ.

A number of detailed studies remain to be done. For example, it is clear that the polarization will help to differentiate between the four models considered here. Also we have only considered the statistical uncertainties, the systematic uncertainties

in measurements of Bhabha scattering could be significant and need to be included in future more realistic studies.

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